The design and implementation of a benchmark based on an iterated post-selective protocol

Adrian Ortega

Universidad de Guadalajara, Mexico

May 2025

The team, collaborators and companies providing qc time

Wigner RCP - Hungary

T. Kiss

O. Kálmán

Z. Udvarnoki

Duke University - USA

N. Linke

A. Green

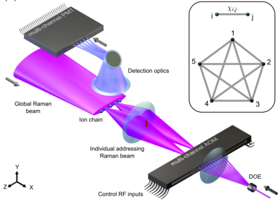
E. Mossman

Universidad de Guadalajara

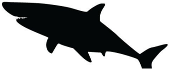
Y. Alonso

ION TRAP QC

(a)



IBM Q™



Current cloud enabled quantum computing

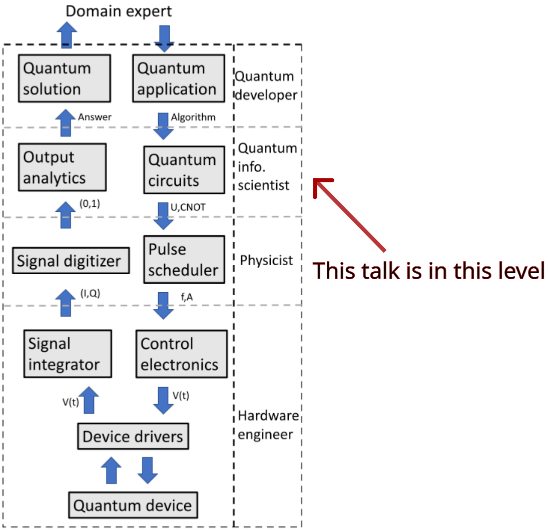


Figure taken from Proceedings of the IEEE 108 (2019).

Quantum benchmark

- ☞ Why a quantum benchmark? How to design a quantum benchmark?
- ☞ Implementation: Decompositions. Choosing the protocol. What to measure?
- ☞ Results

Quantum benchmark: Why?

We have imperfect, highly noisy quantum computers (NISQ-era), so what to do with them? How to decide:

- Which technology is more promising (investment $$$)?
- Is a single physical system (e.g. superconducting qubits, trapped ions, photonic chips) *the* universal system? Is it that a technology is suitable for a specific task?
- Which types of tasks are worth investigating (inside the current qc-capabilities)?
- (etc.)

Quantum benchmark: Why?

An early type of benchmark/testing can help
to decide these matters!

- Ok, but how to?
- Is a single number enough? In general no! Not even preferred in classical benchmarking.
- What are the characteristics of a good benchmark? [Maybe we can use what is already known in classical benchmarking...]

Quantum benchmark: How to?

The current research divides roughly into two categories:

Hallmark protocols

(QC circuits or QI protocols, Quantum chemistry)

Randomized benchmarking

(QV, Mirror circuits)

Our testing/benchmarking fits in the first set.

Quantum benchmark: How to?

Focusing in a *performance benchmark* (a benchmark in general also include security or energy-consumption metrics), for which performance of NISQ systems is usually investigated, it must fulfill the following characteristics[1]:

- The workload has to be representative of real-world applications.
- Exercise all critical services provided by the platforms (for a qc, both hardware and software).
- Not be tuned/optimized for a specific product [or likewise to tune a device for a specific task, i.e. QV and Honeywell [2]].
- Generate reproducible results.
- Not have any inherent scalability limitations [QV is not scalable!].

[1] M. Vieira, H. Madeira, K. Sachs, and S. Kounev. Resilience benchmarking. page 283. Springer, 1 edition, 2012.

[2] https://www.honeywell.com/us/en/news/2021/07/honeywell-sets-another-record-for-quantum-computing-performance

The UnitaryForge package

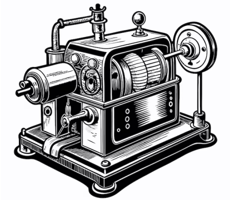
- Quantum circuits are not written in general as a combination of one-qubit gates and CNOTs.
- A reliable, agnostic, decomposition is needed.
- The UnitaryForge package aims to the solution of this problem.
- Arbitrary, entangling two-qubit gate U ($\in \mathrm{SU}(4)$)$\rightarrow$ One-qubit gates and CNOTs.

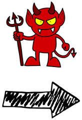
Collab./Developer:
Y. Alonso

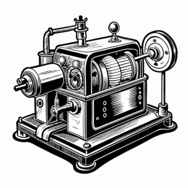
github.com/Yeas3/UnitaryForge
Deployment soon!

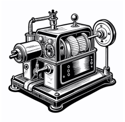
Quantum benchmark: Decompositions

Get $\vec{k} \in \mathbb{R}^3$ | Reorder, rewrite | Factorize









Khaneja-Glaser

$X = (A_0 \otimes A_1) \exp\{-i\vec{k}\cdot\vec{\Sigma}\}(B_0 \otimes B_1)$,
$k_j \in \mathbb{R}$,
$\Sigma_j = (\sigma_j \otimes \sigma_j)$ and
$A_i, B_i \in SU(2)$ $(i = 0, 1)$

PRA 63, 032308 (2001)

Vidal-Dawson

$\pi/4 \geq k'_x \geq k'_y \geq |k'_z| > 0 \rightarrow \times 3$ CNOTs,
$k'_x \geq k'_y > 0, k'_z = 0 \rightarrow \times 2$ CNOTs,
$\vec{k}' = (\pi/4, 0, 0)^T \rightarrow \times 1$ CNOT
+1q gates

PRA 69, 010301 (2004)

Cayley

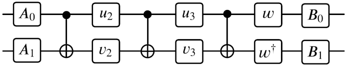
$M^\dagger(\mathcal{C} \otimes \mathcal{D})M \in \mathrm{SO}(4) \rightarrow \mathcal{C}, \mathcal{D} \in \mathrm{SU}(2)$

IEEE Trans. Rob. 30, 1037 (2014)

```
U= [[ 0.12787+0.55181j -0.19527-0.59726j  0.29605+0.19217j  0.34233-0.20624j]
 [-0.12972+0.06977j -0.20892-0.2803j   0.34759-0.3652j  -0.351   +0.69188j]
 [-0.54968+0.19556j -0.63534+0.22603j -0.15539+0.38409j -0.17914-0.03313j]
 [-0.29599-0.47902j -0.08102+0.147j    0.6661 +0.0766j   0
```

```
Algoritmo KG:

A0= [[ 0.41141+0.78943j -0.21083+0.40384j]
 [ 0.21083+0.40384j  0.41141-0.78943j]]

A1= [[-0.96144+0.1404j   0.17528+0.15877j]
 [-0.17528+0.15877j -0.96144-0.1404j ]]

B0= [[ 0.46875+0.12124j  0.74859-0.45297j]
 [-0.74859-0.45297j  0.46875-0.12124j]]

B1= [[-0.46875-0.26441j  0.54108-0.64621j]
 [-0.54108-0.64621j -0.46875+0.26441j]]

k= [-0.41389971  0.06501331 -0.17815323]
k0= 0.7853981634
```

```
Descomposición en cnots:

Número de cnots= 3

u2= [[-0.25669+0.65887j -0.25669+0.65887j]
 [ 0.25669+0.65887j -0.25669-0.65887j]]

v2= [[0.75155-0.65967j 0.      +0.j      ]
 [0.      +0.j      0.65967-0.75155j]]

u3= [[0.-0.70711j 0.-0.70711j]
 [0.-0.70711j 0.+0.70711j]]

v3= [[0.98417+0.17721j 0.      +0.j      ]
 [0.      +0.j      0.98417-0.17721j]]

w= [[0.70711+0.j       0.      -0.70711j]
 [0.      -0.70711j 0.70711+0.j      ]]
```



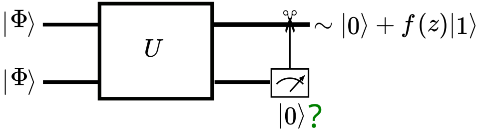
Quantum benchmarks: The protocol

Quantum state matching using a superattractive map

[O. Kálmán and T. Kiss, Phys. Rev. A 97, 032125 (2018)]

One step, 2 qubits. Initial state:

$$|\Phi\rangle \propto |0\rangle + z|1\rangle \rightarrow |\Psi_0\rangle = |\Phi\rangle \otimes |\Phi\rangle.$$



U is an entangling unitary and $f(z)$ is a quadratic rational complex function:

$$f(z) = \frac{a_0 + a_1 z + a_2 z^2}{b_0 + b_1 z + b_2 z^2}.$$

Collabs:

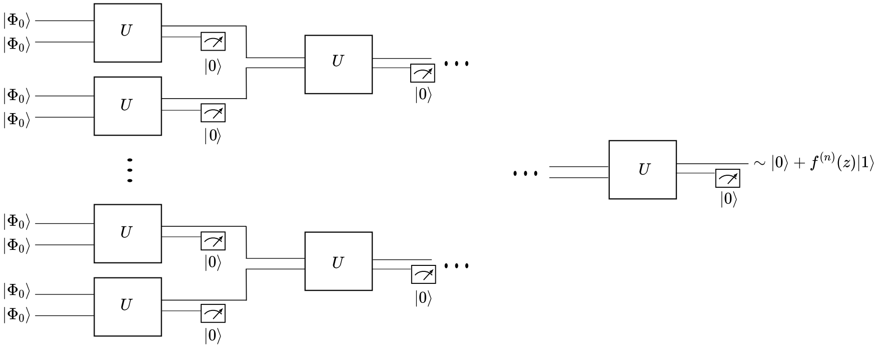
T. Kiss, O. Kálmán & Z. Udvarnoki



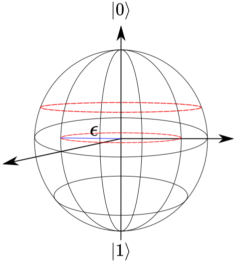
Full story: Phys. Scr. 98, 024006 (2023)

Quantum benchmarks: The protocol

After *n*-steps:



Quantum benchmarks: The protocol of quantum state matching



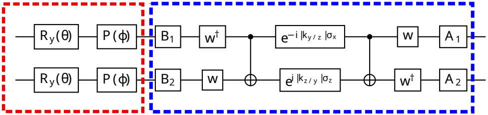
Transformation (not ϕ dependant!)

$$f_\epsilon(z) = \frac{z^2}{\epsilon}, \quad z = e^{i\phi}\tan(\theta/2)$$

achieved using

$$U_\epsilon = \begin{pmatrix} \epsilon & -\frac{1}{\sqrt{2}}\sqrt{1-\epsilon^2} & \frac{1}{\sqrt{2}}\sqrt{1-\epsilon^2} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \\ \sqrt{1-\epsilon^2} & \frac{1}{\sqrt{2}}\epsilon & -\frac{1}{\sqrt{2}}\epsilon & 0 \end{pmatrix}.$$

State preparation



U_ϵ

$w = (\mathbb{1} - i\sigma_x)/\sqrt{2}$, see [G. Vidal and C. M. Dawson, Phys. Rev. A, 69:010301 (2004)].

Characteristics:

- Super-attractive = Fast convergence.
- Orthogonalizing map for two states.
- n-steps uses 2^n qubits (good for testing purposes).
- **No classical overhead** (c.f. Quantum Volume benchmark).

– Goal: Benchmarking of quantum computers using the quantum state matching protocol.

– Errors in the measurement: statistical and device specific. A simple approach is to consider

$$\Delta_S/\sqrt{M} + \Delta_D,$$

Quantum benchmarks: What to measure?

M: total number of experiments, Δ_S: statistical noise, Δ_D: device noise[3].

Success probability of the post-selected state

$$\begin{aligned} p_s &= P_{|0\rangle} + P_{|1\rangle} \\ &= \epsilon^{2^{n+1}-2} \cos(\theta/2)^{2^{n+1}} + \sin(\theta/2)^{2^{n+1}}, \end{aligned}$$

at step n "filters out" the statistical error if we consider the interval

$$p_{s,\pm}^{(e)} = p_s \pm m\sigma,$$

for m an integer and $\sigma = \sqrt{p_s(1-p_s)/M}$.

[3]A. Cornelissen, et. al. arxiv.2104.10698

Quantum Benchmark: Results I - Initial states

Influence of the initial (random) state for two different qcs (QVs)

Errors

Masking of errors

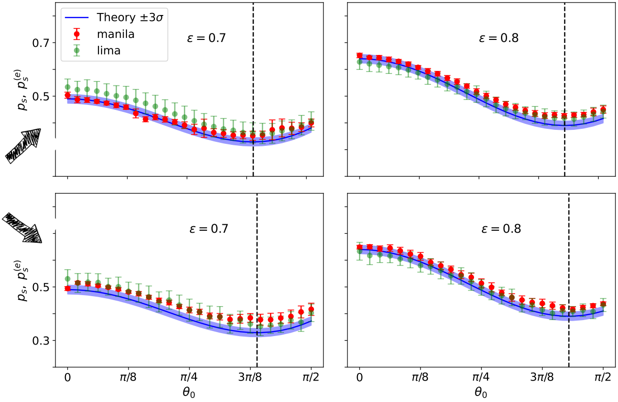


Figure: One step, manila 32QV and lima 8QV. Upper row: deterministic ϕ_0 initial states. Bottom row: Random ϕ_0 initial states.

More details: Phys. Scr. 98, 024006 (2023).

Quantum benchmark: Results II - Coherent errors

The following set of errors are related with the implementation of gates in a certain qc, known as miss-rotations. The CNOT gate is the main source of such errors. In the IBM devices, the two-qubit interaction is the cross-resonance gate (Λ is the error in the rotation)

$$U(1/2 + \Lambda) = \exp(-i(1/2 + \Lambda)\pi Z_1 X_2/2)$$

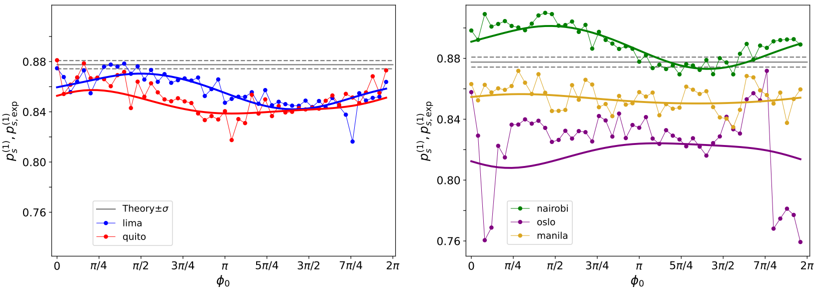
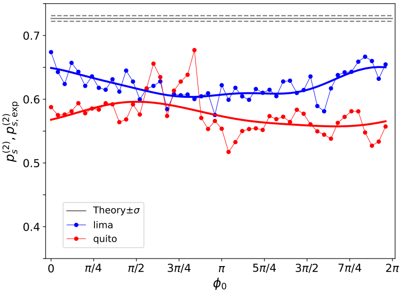
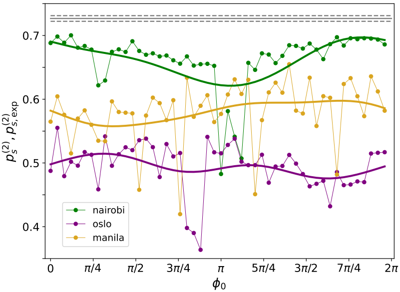
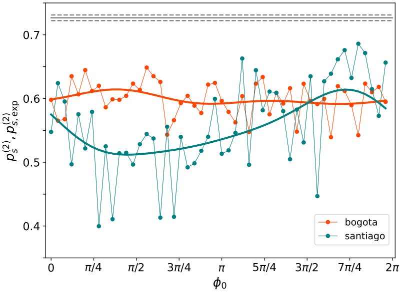


Figure: Success probability p_s as a function of ϕ_0 for $\theta = \pi/8$ and one step of the protocol.

Quantum benchmark: Results II - Coherent errors







Full story:
`https://arxiv.org/abs/2410.07056`

(2024)

Figure: Success probability p_s as a function of ϕ_0 for $\theta = \pi/8$ and one step of the protocol.

Quantum benchmark: Results III - Statistical metrics

The performance of a quantum circuit in a certain qc is measured with the metrics:

$$F^{(n)} = 1 - \frac{|\bar{p}^{(n)}_{s,\mathrm{exp}} - p^{(n)}_s|}{p^{(n)}_s}\,, \qquad (1)$$

$$S^{(n)} = \frac{\sigma^{(n)}_{s,\mathrm{exp}}}{\sigma^{(n)}_s}\,, \qquad (2)$$

- $F^{(n)}$ estimates how much all measurements are close to the theoretical value of p_s.
- $S^{(n)}$ measures the "dispersion" in experiment measurements with respect to the pure statistical noise. Captures the total "noise" that affects the qc.

Quantum benchmark: Results III - Statistical metrics

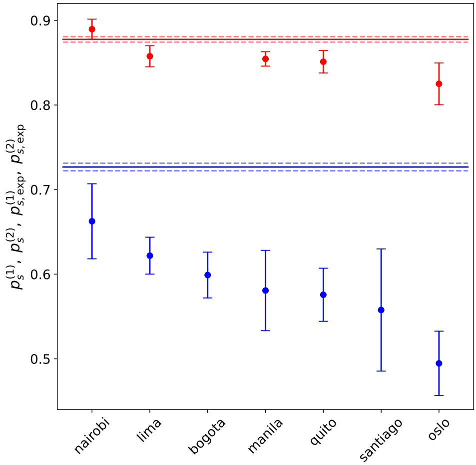
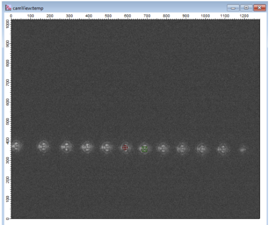


Figure: Sucess probability $\overline{p}_{s,\mathrm{exp}}^{(n)}$ averaged over the initial angle ϕ_0 and its respective variance, with $n = 1, 2$ steps.

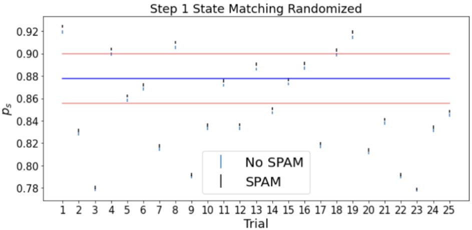
Device	QV	$F^{(1)}$	$S^{(1)}$
nairobi	32	0.986	3.609
lima	8	0.977	3.812
manila	32	0.974	2.599
quito	16	0.970	4.041
oslo	32	0.940	7.544

Device	QV	$F^{(2)}$	$S^{(2)}$
nairobi	32	0.912	9.953
lima	8	0.856	4.893
bogota	32	0.824	6.077
manila	32	0.799	10.650
quito	16	0.792	7.045
santiago	32	0.767	16.204
oslo	32	0.681	8.539

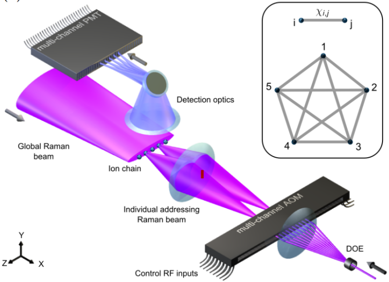
Quantum benchmark: Results IV - Ion traps (in progress)

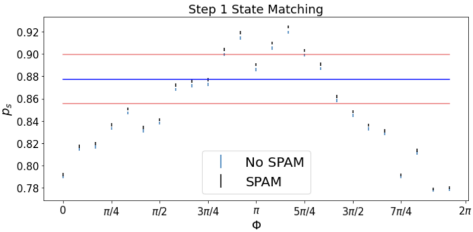


← Real ions!



(a)







Collabs:

N. Linke, A. Green & E. Mossman

Outlook

Advantages of our algorithm:

- Highly structured circuit (fast transpilation).
- No classical overhead [c.f. Quantum Volume].
- 2^n qubits tested [Testing advantage!]
- Opportunity to test any pair of basis states $\left|i_0 i_1 \ldots i_{2^n-1}\right\rangle$, $\left|j_0 j_1 \ldots j_{2^n-1}\right\rangle$ [albeit SWAP gates must be introduced...].

Experiments:

- Time variations via the scanning of (ϕ_0, θ_0) and the invariance of $f(z)$ over ϕ_0.
- Dependence of the initial states. Random initial states are better?
- Equal Quantum Volume does not imply equal application of $f(z)$ (!).

¡Gracias por su atención!